

**Reassessment of the Collins Mechanism for Single-spin Asymmetries and the behavior of  $\Delta d(x)$  at large  $x$ .**

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It is shown that the Collins mechanism explanation of the transverse single-spin asymmetries in  $p^\uparrow p \rightarrow \pi X$  leads to a transversely polarized  $d$  quark density  $\Delta_T d(x)$  which violates the Soffer bound when one uses several standard forms for the longitudinally polarized  $d$  quark density  $\Delta d(x)$  obtained from polarized deep inelastic scattering. Imposition of the Soffer bound with these  $\Delta d(x)$  yields results in hopeless disagreement with the data. Remarkably, imposition of the Soffer bound, but using parametrizations of  $\Delta d(x)$  that respect the PQCD condition  $\Delta q(x)/q(x) \rightarrow 1$  as  $x \rightarrow 1$ , leads to an excellent fit to most of the data. The implications for the polarized DIS neutron longitudinal asymmetry  $A_1^n$  at large  $x$  are dramatic.

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**I. INTRODUCTION**

One of the major challenges to the QCD-parton model is the explanation of the large (20-40%) single-spin asymmetries found in many semi-inclusive hadron-hadron reactions, of which the most dramatic are the polarization of the lambda in  $pp \rightarrow \Lambda X$  and the asymmetry under reversal of the transverse spin of the proton in  $p^\uparrow p \rightarrow \pi X$ . The challenge arises from the fact that in the standard approach the basic “two parton  $\rightarrow$  two parton” reactions involved in the perturbatively treated hard part of the scattering do not possess this kind of asymmetry.

Already some time ago, Efremov and Teryaev [1] suggested a mechanism for these asymmetries utilizing “three parton  $\rightarrow$  two parton” amplitudes for the hard scattering. This, however, necessitates the introduction of a new unknown soft two-parton density, namely the correlated probability of finding in the polarized proton a quark with momentum fraction  $x_1$  and a gluon with fraction  $x_2$ . This quark-gluon correlator contains the dependence on the transverse spin of the proton. A fully consistent application of the approach has not yet been carried out, though a significant step in the direction has been taken recently by Sterman and Qiu [2].

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Some time ago Sivers [3] and Collins [4] suggested mechanisms for the asymmetries, which are within the framework of the standard “two parton  $\rightarrow$  two parton” picture, but in which the transverse momentum  $\mathbf{k}_T$  of the quark in the hadron plays an essential role. Sivers introduces a parton density which depends on the transverse spin  $\mathbf{S}_T$  of the proton in the form  $\mathbf{S}_T \cdot (\mathbf{k}_T \times \mathbf{p})$  where  $\mathbf{p}$  is the momentum of the polarized proton. However such a structure is forbidden by time reversal invariance [4]. Nonetheless, on the grounds that the time-reversal argument could be invalidated by initial or final state interactions, Anselmino, Boglione and Murgia [5,6] have applied the Sivers mechanism to  $p^\uparrow p \rightarrow \pi X$  and shown that a very good fit to the data at large  $x_F$  can be achieved. The problem is that this approach raises a fundamental question: if the initial and final state interactions are important then it is hard to see why the underlying factorization into hard and soft parts is valid. (The advantage of the quark-gluon correlator approach is that it effectively provides initial and final state interactions calculable at the parton level). In the Collins mechanism the asymmetry arises in the fragmentation of a polarized quark into a pion with transverse momentum  $p_T$ , described by a fragmentation function of the form  $\Delta^N D(z, p_T)$  (see Section II) which is convoluted with the transverse spin dependent parton density  $\Delta_T q$ , about which nothing is known experimentally. This structure is not in conflict with time reversal invariance. The definition of transverse spin dependent parton densities and possible methods of determining them have been studied by Artru and Mekhfi [7], Cortes, Pire and Ralstone [8] and Jaffe and Ji [9]. Note that in the latter they are referred to as “transversity distributions” and that notation differs amongst all these papers.

An estimate of the size of the Collins effect was first made by Artru, Czyzewski and Yabuki [10], but more recently Anselmino, Boglione and Murgia [6] have demonstrated that an excellent fit to the data on  $p^\uparrow p \rightarrow \pi X$  can be obtained with the Collins mechanism. However their fit is problematic since the  $\Delta_T q$ ’s used in the fit violate the Soffer bound [11]

$$|\Delta_T q(x)| \leq \frac{1}{2}[q(x) + \Delta q(x)] \quad (1)$$

at large  $x$  when  $\Delta q(x)$ , the usual longitudinal polarized parton density, is taken from any of the standard parametrizations of the longitudinal and unpolarized densities:

- GRSV-GRV = Glück, Reya, Stratmann and Vogelsang [12] + Glück, Reya, Vogt [13],
- GS-GRV = Gehrman and Stirling [14] + Glück, Reya, Vogt [13],
- LSS-MRST = Leader, Sidorov and Stamenov [15] + Martin, Roberts, Stirling and Thorne [16].

Note that in the above parametrizations the polarized densities are linked to particular parametrizations of the unpolarized densities, as indicated.

The key point is that the  $\pi^-$  data demand a large magnitude for  $\Delta_T d$  at large  $x$ , whereas  $\Delta d(x)$  is almost universally taken negative for all  $x$ , thereby making the Soffer bound much more restrictive for the  $d$  than for the  $u$  quark.

This raises an intriguing question. There is an old perturbative QCD argument [17] that strictly in the limit  $x \rightarrow 1$

$$\frac{\Delta q(x)}{q(x)} \rightarrow 1, \quad (2)$$

which would imply that  $\Delta d(x)$  has to change sign and become positive at large  $x$ . The polarized DIS data certainly require a negative  $\Delta d(x)$  in the range  $0.004 < x \leq 0.75$  where data exist, but there is no reason why  $\Delta d(x)$  should not change sign near or beyond 0.75. Indeed there is a parameterization of the parton densities by Brodsky, Burkhardt and Schmidt

[BBS] [18] which has this feature built into it. The original BBS fit is not really competitive since evolution in  $Q^2$  was not taken into account, but a proper QCD fit based on the BBS parameterization was shown by Leader, Sidorov and Stamenov [19] to give an adequate fit to the polarized DIS data.

In this paper we address the question of the correct use of the Collins mechanism in which the Soffer bound is respected. We find that it is impossible to get a good fit to the  $\pi^\pm$  data when the magnitude of  $\Delta_T d$  is controlled by (1) in which  $\Delta d(x)$  from any of the standard forms given above is used. On the contrary, and most surprisingly, we find that parametrizations in which  $\Delta d(x)/d(x) \rightarrow 1$  as  $x \rightarrow 1$  allow a  $\Delta_T d(x)$  that leads to an excellent fit to most of the pion data.

In Section II we briefly describe the Collins mechanism, and present our results in Section III. Conclusions follow in Section IV.

## II. THE MODEL

As mentioned in the Introduction, we require the Soffer inequality, Eq.(1), to be respected by the distribution functions  $\Delta_T u$  and  $\Delta_T d$  determined by our fit. Besides, the positivity constraint  $\Delta^N D(z) \leq 2 D(z)$  must hold, since  $D(z) = \frac{1}{2} [D^\uparrow(z) + D^\downarrow(z)]$  and  $\Delta^N D(z) = [D^\uparrow(z) - D^\downarrow(z)]$ . Therefore, the parametrizations are set so that these conditions are automatically fulfilled, in the following way. First we build a simple function of the form  $x^a(1-x)^b$  (or  $z^\alpha(1-z)^\beta$ , as appropriate), where the powers  $a, b$  or  $\alpha, \beta$  are  $\geq 0$ , and we divide by their maximum value. By allowing an extra multiplicative constant factor to vary from  $-1$  to  $1$ , we obtain a function which, in modulus, will never be larger than  $1$ . Then we parameterize  $\Delta_T q(x)$  and  $\Delta^N D(z)$  by multiplying the functions we built by the constraint given by the Soffer inequality or the positivity limit. In this way we make sure that the bounds are never broken.

For the transversity distribution functions we set

$$\Delta_T u(x) = N_u \frac{x^a(1-x)^b}{\frac{a^a b^b}{(a+b)^{a+b}}} \left\{ \frac{1}{2} [u(x) + \Delta u(x)] \right\}, \quad (3)$$

$$\Delta_T d(x) = N_d \frac{x^c(1-x)^d}{\frac{c^c d^d}{(c+d)^{c+d}}} \left\{ \frac{1}{2} [d(x) + \Delta d(x)] \right\}, \quad (4)$$

with  $|N_{u,d}| \leq 1$ . Here  $q(x)$  and  $\Delta q(x)$  are the whole distribution functions, i.e. they contain valence and sea contributions (but this is irrelevant at large  $x$  since there the contribution of the sea is negligible). As in the previous calculation, only  $u$  and  $d$  contributions are taken into account in the polarized proton, so that

$$\Delta_T \bar{u}(x) = \Delta_T \bar{d}(x) = \Delta_T s(x) = \Delta_T \bar{s}(x) = 0. \quad (5)$$

For the functions  $q(x)$  and  $\Delta q(x)$  we use, for comparison, the “standard” parton parametrizations mentioned in Section I and two further parametrizations, one due to Brodsky, Burkhardt and Schmidt (BBS) [18] which ignores  $Q^2$ -evolution, and a more consistent version of this, due to Leader, Sidorov and Stamenov (LSS)<sub>BBS</sub> [19] which includes the  $Q^2$ -evolution. These will be explained in more detail in Section III.

For the fragmentation function we have

$$\Delta^N D(z) = N_F \frac{z^\alpha(1-z)^\beta}{\frac{\alpha^\alpha \beta^\beta}{(\alpha+\beta)^{\alpha+\beta}}} \left[ 2 D(z) \right], \quad (6)$$

with  $|N_F| \leq 1$ . Here we take into account only valence contributions, so that isospin symmetry and charge conjugation give

$$\Delta^N D_{\pi^+}^u = \Delta^N D_{\pi^-}^d = \Delta^N D(z), \quad (7)$$

$$\Delta^N D_{\pi^+}^{\bar{u}} = \Delta^N D_{\pi^-}^{\bar{d}} = 0 \quad (8)$$

and

$$\Delta^N D_{\pi^0}^u = \Delta^N D_{\pi^0}^d = \Delta^N D_{\pi^0}^{\bar{u}} = \Delta^N D_{\pi^0}^{\bar{d}} = \frac{1}{2} \Delta^N D(z). \quad (9)$$

Notice that  $\Delta^N D$  is, in fact, a function of the intrinsic transverse momentum  $\mathbf{p}_T$ .  $\Delta^N D_{h/a^\uparrow}(z, \mathbf{p}_T)$  is defined as the difference between the number density of hadrons  $h$ , a pion in our case, with longitudinal momentum fraction  $z$  and transverse momentum  $\mathbf{p}_T$ , originating from a transversely polarized parton  $a$  with spin either  $\uparrow$  or  $\downarrow$ , respectively

$$\begin{aligned} \Delta^N D_{h/a^\uparrow}(z, \mathbf{p}_T) &\equiv \hat{D}_{h/a^\uparrow}(z, \mathbf{p}_T) - \hat{D}_{h/a^\downarrow}(z, \mathbf{p}_T) \\ &= \hat{D}_{h/a^\uparrow}(z, \mathbf{p}_T) - \hat{D}_{h/a^\uparrow}(z, -\mathbf{p}_T), \end{aligned} \quad (10)$$

where the second line follows from the first one by rotational invariance. Details on the integration over the transverse momentum  $\mathbf{p}_T$  and its dependence on  $z$  are given in Ref. [6] (see Eqs. (17) and (19)). The unpolarized fragmentation function for pions,  $D(z)$ , is taken from Binnewies et al. [20].

With these ingredients we are now ready to calculate, in complete analogy with Ref. [6], the  $p^\uparrow p \rightarrow \pi X$  single spin asymmetry

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}. \quad (11)$$

Here

$$\begin{aligned} d\sigma^\uparrow - d\sigma^\downarrow &= \sum_{a,b,c,d} \int \frac{dx_a dx_b}{\pi z} d^2 \mathbf{p}_T \\ &\times \Delta_T q^a(x_a) q^b(x_b) \Delta_{NN} \hat{\sigma}^{ab \rightarrow cd}(x_a, x_b, \mathbf{p}_T) \Delta^N D_{\pi/c}(z, \mathbf{p}_T), \end{aligned} \quad (12)$$

where

$$\Delta_{NN} \hat{\sigma}^{ab \rightarrow cd} = \frac{d\hat{\sigma}^{a^\uparrow b \rightarrow c^\uparrow d}}{d\hat{t}} - \frac{d\hat{\sigma}^{a^\uparrow b \rightarrow c^\downarrow d}}{d\hat{t}}, \quad (13)$$

and

$$\begin{aligned} d\sigma^\uparrow + d\sigma^\downarrow &= 2 d\sigma^{unp} = 2 \sum_{a,b,c,d} \int \frac{dx_a dx_b}{\pi z} \\ &\times q^a(x_a) q^b(x_b) \frac{d\hat{\sigma}^{ab \rightarrow cd}}{d\hat{t}}(x_a, x_b) D_{\pi/c}(z). \end{aligned} \quad (14)$$

All details about the calculation can be found in Ref. [6]. The relation between the above notation and that of [6] is :  $q^a(x) = f_{a/p}(x)$  and  $\Delta_T q^a(x) = P^{a/p^\uparrow} f_{a/p}(x)$ .

### III. RESULTS

We start by running two fits to the *E704* experimental data [21] using the popular GS [14] polarized densities in conjunction with the GRV [13] unpolarized densities, and the latest LSS [15] polarized densities in conjunction with the MRST [16] unpolarized densities. It should be noted that the 1996 analysis of Gehrmann and Stirling was done prior to the publication of a great deal of new, high precision data on polarized DIS, whereas the Leader, Sidorov and Stamenov analysis includes all of the present world data. Fig. 1 shows the complete failure to fit the data when the Soffer bound is implemented using the GS-GRV parametrizations ( $\chi^2/DOF = 25$  !). The corresponding transverse densities  $\Delta_T u(x)$  and  $\Delta_T d(x)$  are shown in Fig. 2. In this fit only  $|N_F| = 1$  is possible and Fig. 2 corresponds to  $N_F = -1$ . The sign is discussed later.

A somewhat better picture emerges when using the LSS-MRST results to implement the Soffer bound: Fig. 3. The fit looks reasonable out to  $x_F = 0.4$  but fails beyond that. One finds  $\chi^2/DOF = 6.12$ . The transverse densities are shown in Fig. 4, where the curves correspond to negative  $N_F$ .

Note that in both these fits one finds  $\alpha = \beta = 0$  in Eqn. (6), showing that the magnitude of  $\Delta^N D(z)$  is maximized at each  $z$ -value.

The reason for the failure of GS-GRV case and for the relative success of LSS-MRST can be understood by observing in Fig. 5 that the Soffer bound on  $\Delta_T d(x)$  is *much* more restrictive at large  $x$  in the GS-GRV case. Comparison of Fig. 5 with Fig. 6 also indicates the source of the problem. The asymmetries for  $\pi^\pm$  are of roughly equal magnitude whereas the Soffer bound restrictions are much more severe for the  $d$  quark as a consequence of  $\Delta d(x)$  being negative for all  $x$ .

This suggests an intriguing possibility. The polarized DIS data only exist for  $x \leq 0.75$  and there is really very little constraint from DIS on the  $\Delta q(x)$  for  $x$  near to and beyond this value. At the same time there are perturbative QCD arguments [17] which suggest that

$$\frac{\Delta q(x)}{q(x)} \rightarrow 1 \quad \text{as } x \rightarrow 1 \quad (15)$$

and, indeed, even more precisely, that

$$q(x) - \Delta q(x) \propto (1-x)^2 q(x) \quad \text{as } x \rightarrow 1. \quad (16)$$

This constraint is almost universally ignored in parameterizing the  $\Delta q(x)$ , on the grounds that (16) is incompatible with the evolution equations. But this is a “red herring” since the evolution equations do not hold in the region where (16) is valid, approaching the border of the exclusive region.

The imposition of (15) is exactly what we need for  $\Delta q(x)$  to change sign and become positive at large  $x$ , thereby diminishing the restrictive power of the Soffer bound on  $\Delta_T d(x)$ .

In fact there does exist a parametrization of the  $\Delta q(x)$  which respects (15) and (16), namely that of Brodsky, Burkhardt and Schmidt (BBS) [18]. Unfortunately BBS did not include any  $Q^2$ -evolution where determining the numerical values of their parameters from the DIS data, so their fit is not really adequate.

However, Leader, Sidorov and Stamenov [19] made an extensive study, using the BBS functional forms, but including  $Q^2$ -evolution, and found a very good fit (LSS)<sub>BBS</sub> to the polarized DIS data available in 1997.

It can be seen in Fig. 5 that the Soffer bound on  $\Delta_T d(x)$  is much less restrictive for the BBS case and that the (LSS)<sub>BBS</sub> bound is rather similar to that of the LSS-MRST case, but is less restrictive for  $x \geq 0.7$ . It is important to realize that although the  $\Delta_T d(x_a)$  needed in (12) are tiny for such large values of  $x_a$ , this is compensated for by the fact that large  $x_F$  then demands very small  $x_b$ , where the unpolarized densities grow very large.

	GS-GRV	LSS-MRST	BBS	(LSS) <sub>BBS</sub>
$N_F N_u$	-0.43	-0.73	-0.54	-0.49
$N_F N_d$	1.00	1.00	0.88	0.91
$a$	4.33	3.03	3.17	3.46
$b$	0.00	0.00	0.00	0.00
$c$	0.00	3.48	3.57	3.32
$d$	0.00	0.76	0.00	0.00
$\chi^2/DOF$	25	6.12	1.45	2.41

TABLE I. Parameters determined by the fit in the four different parameterization schemes and the corresponding  $\chi^2/DOF$ .

Note that the relative signs of  $N_u$  and  $N_d$  are opposite, but their absolute signs are not determined since, in principle,  $N_F$  can be positive or negative. However, if one uses an  $SU(6)_F$  wave function for the proton, one finds  $\Delta_T u$  positive and  $\Delta_T d$  negative, so it seems reasonable to hypothesize that  $N_u > 0$  and  $N_d < 0$ . For this reason we have chosen  $N_F$  to be negative in the above. Note that  $N_u$  and  $N_d$  are not a direct measure of the magnitudes of  $\Delta_T u$  and  $\Delta_T d$ . Their role is linked specifically to the Soffer bound. The relative behavior of  $\Delta_T u$  and  $\Delta_T d$  can be seen in Figs. 2, 4, 9, 10.

Indeed, as expected, we find that a significantly better fit to the asymmetry data is achieved using the BBS and the (LSS)<sub>BBS</sub> parametrizations, with  $\chi^2/DOF = 1.45$  and  $\chi^2/DOF = 2.41$  respectively. As can be seen in Fig. 7 and 8. The curves reproduce the trends in the data right out to  $x_F \sim 0.7$ . Figs. 9 and 10 show how similar the allowed ranges of transverse polarized densities are in the two cases. In Fig. 9,  $0.88 \leq |N_F| \leq 1$ , whereas in Fig. 10  $0.91 \leq |N_F| \leq 1$ . As before the curves correspond to negative  $N_F$ .

The parameter values for all the parametrizations are shown in Table 1, where it should be recalled  $|N_F|$ ,  $|N_u|$  and  $|N_d| \leq 1$ .

#### IV. CONCLUSIONS

We have demonstrated that the Collins mechanism is able to explain much of the data on the transverse single spin asymmetries in  $p^\uparrow p \rightarrow \pi X$ , namely the data in the region  $x_F \leq 0.7$ , if, and only if, the longitudinal polarized  $d$ -quark density, which is negative for small and moderate  $x$ , changes sign and becomes positive at large  $x$ . There is hopeless disagreement when using the longitudinal polarized densities due to Gehrmann and Stirling [14], and matters are significantly better when using the most up to date parameterization of Leader, Sidorov and Stamenov [15]. But the most successful fits arise from parametrizations [18,19] which respect the PQCD condition  $\Delta d(x)/d(x) \rightarrow 1$  as  $x \rightarrow 1$ .

For parametrizations of  $\Delta d(x)$  with this property there are interesting consequences in polarized DIS, namely, the neutron longitudinal asymmetry  $A_1^n(x)$  should change sign and tend to 1 as  $x \rightarrow 1$  (see Fig. 11). The region of large  $x$  has hardly been explored in polarized DIS up to the present. Clearly a study of this region might turn out to be very interesting.

There remains, however, the problem of the  $p^\uparrow p \rightarrow \pi X$  data at the largest values of  $x_F$  so far measured, i.e.  $0.7 \leq x_F \leq 0.82$ . It does not seem possible to account for these asymmetries within the framework of the Collins mechanism. On the other hand Qiu and Serman [2], using a “three parton  $\rightarrow$  two parton” amplitude for the hard partonic scattering and a “gluonic pole” mechanism, claim to be able to reproduce the very large asymmetries at  $x_F \sim 0.8$ . However their study must be considered as preliminary, since

it relies on a completely *ad hoc* assumption that the essential new twist-three quark-gluon-quark correlator function  $T_F^{(v)}(x, x)$ , for given flavour  $f$ , is proportional to  $q_f(x)$ , and no attempt is made to fit the detailed  $x_F$ -dependence of the data.

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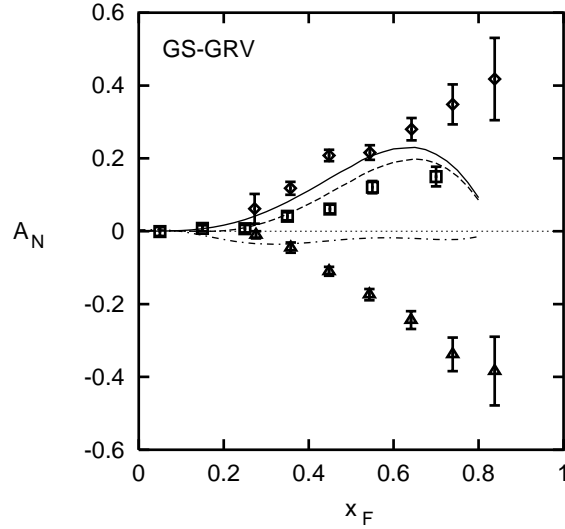


FIG. 1. Single spin asymmetry for pion production in the process  $p^\uparrow p \rightarrow \pi X$  as a function of  $x_F$ , obtained by using the GS-GRV [13,14] sets of distribution functions. The solid line refers to  $\pi^+$ , the dashed line to  $\pi^0$  and the dash-dotted line to  $\pi^-$ .

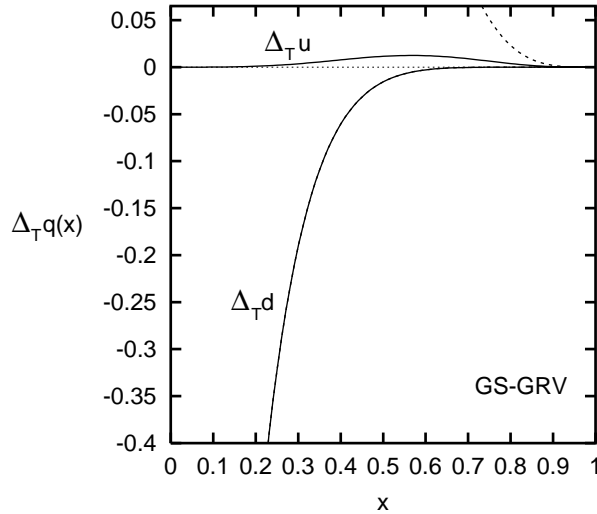


FIG. 2. The distribution functions  $\Delta_T u(x)$  and  $\Delta_T d(x)$ , as obtained by using the GS-GRV [13,14] distribution functions. The dotted lines are the boundaries imposed by the Soffer inequality. For  $\Delta_T d(x)$  the dotted line is invisible since  $\Delta_T d(x)$  completely saturates the Soffer bound in the whole  $x$  region. For discussion of signs, see text.



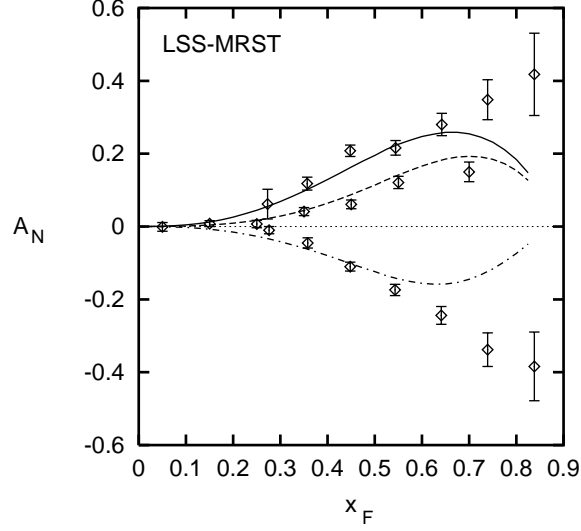


FIG. 3. The single spin asymmetry for pion production in the process  $p^\uparrow p \rightarrow \pi X$  as a function of  $x_F$ , obtained using the LSS-MRST [15,16] distribution functions. The solid line refers to  $\pi^+$ , the dashed line to  $\pi^0$  and the dash-dotted line to  $\pi^-$ .

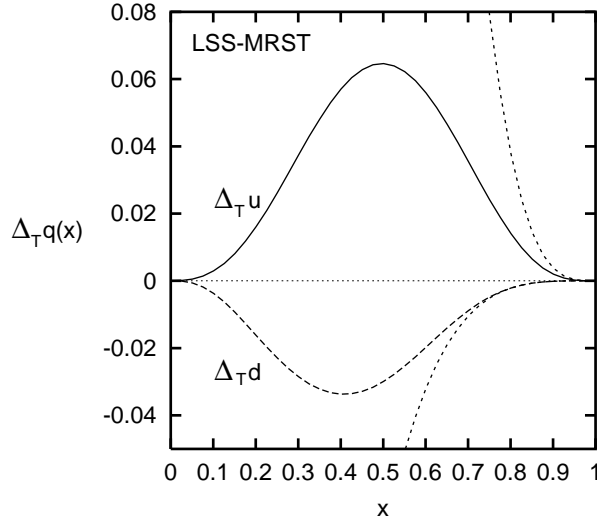


FIG. 4. The distribution functions  $\Delta_T u(x)$  and  $\Delta_T d(x)$  versus  $x$ , as determined by the fit using the LSS-MRST [15,16] distribution functions. The dotted lines are the boundaries imposed by the Soffer inequality. For discussion of signs see text.

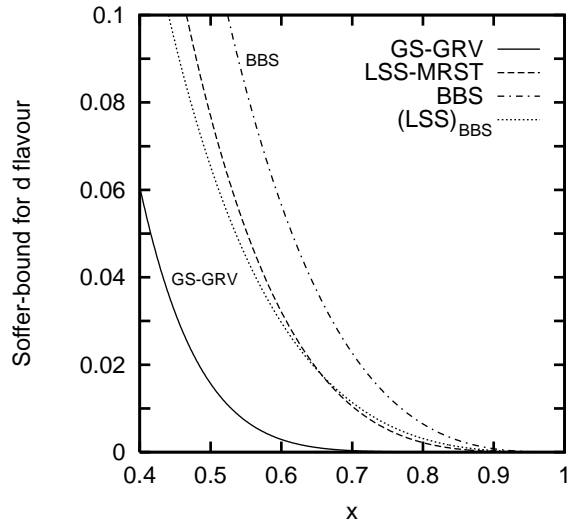


FIG. 5. The boundaries imposed on  $\Delta_T d$  by the Soffer inequality. Note that the GS-GRV distribution functions give a much tighter bound than any of the other parameterization sets.

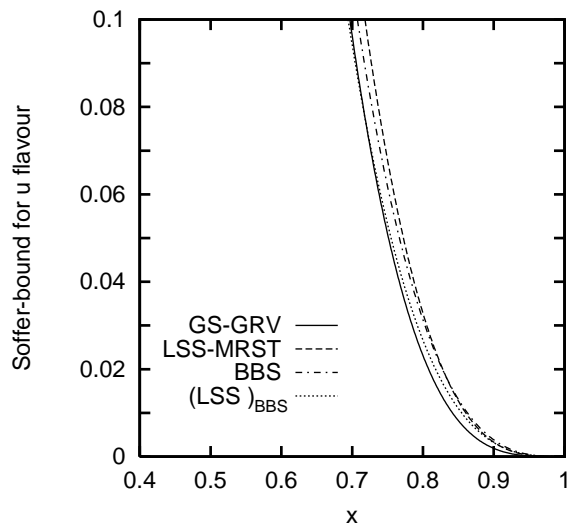


FIG. 6. The boundaries imposed on  $\Delta_T u$  by the Soffer inequality. For the  $u$  flavour the Soffer bound is very similar in each set of parametrizations.

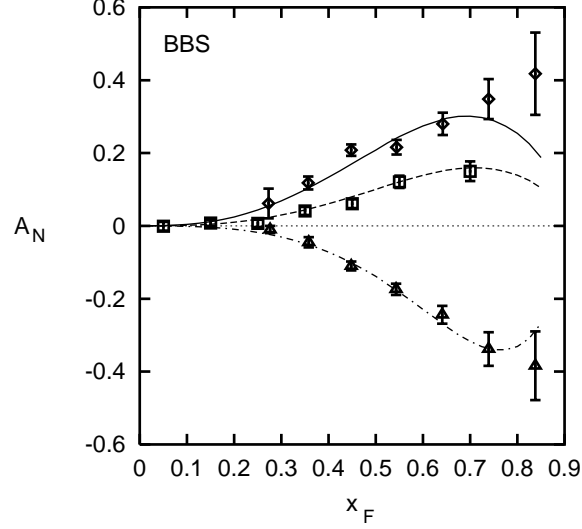


FIG. 7. The single spin asymmetry for pion production in the process  $p^\uparrow p \rightarrow \pi X$  as a function of  $x_F$ , obtained by using the BBS [18] distribution functions. The solid line refers to  $\pi^+$ , the dashed line to  $\pi^0$  and the dash-dotted line to  $\pi^-$ .

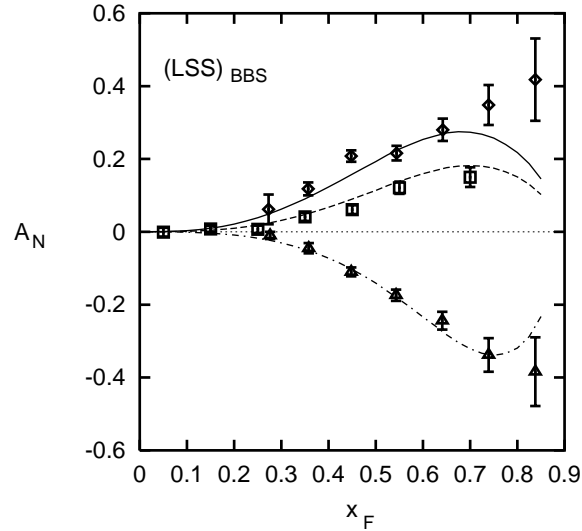


FIG. 8. The single spin asymmetry for pion production in the process  $p^\uparrow p \rightarrow \pi X$  as a function of  $x_F$ , determined by the fit using the  $(LSS)_{BBS}$  [19] distribution functions. The solid line refers to  $\pi^+$ , the dashed line to  $\pi^0$  and the dash-dotted line to  $\pi^-$ .

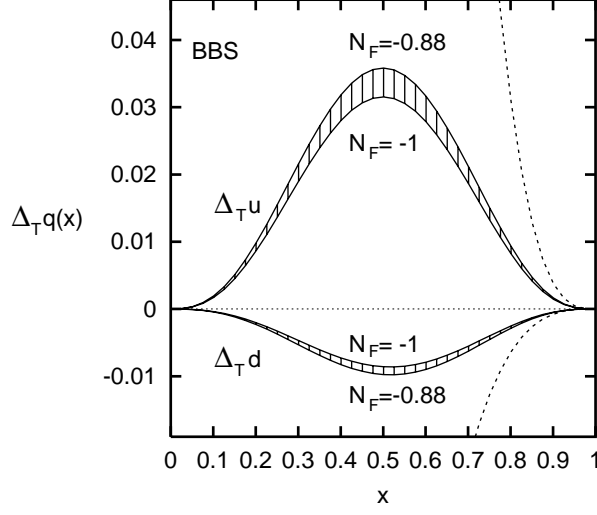


FIG. 9. The allowed range of distribution functions  $\Delta_T u(x)$  and  $\Delta_T d(x)$  versus  $x$ , as determined by the fit using the BBS [18] distribution functions. The dotted lines are the boundaries imposed by the Soffer inequality. For signs see discussion in text.

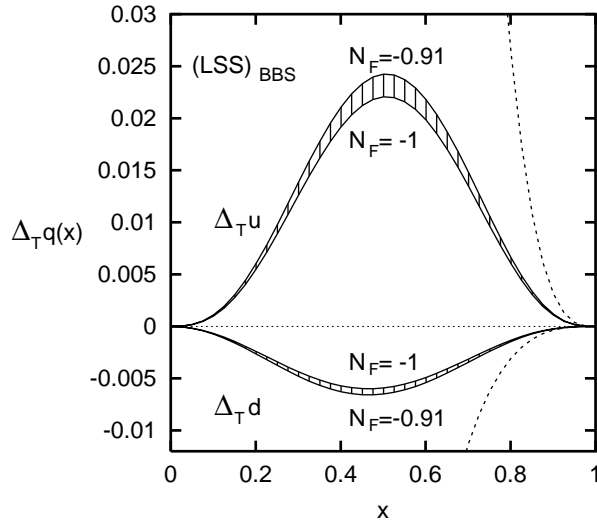


FIG. 10. The range of allowed distribution functions  $\Delta_T u(x)$  and  $\Delta_T d(x)$  versus  $x$ , as determined by the fit using the  $(LSS)_{BBS}$  [19] distribution functions. The dotted lines are the boundaries imposed by the Soffer inequality. For signs see discussion in text.

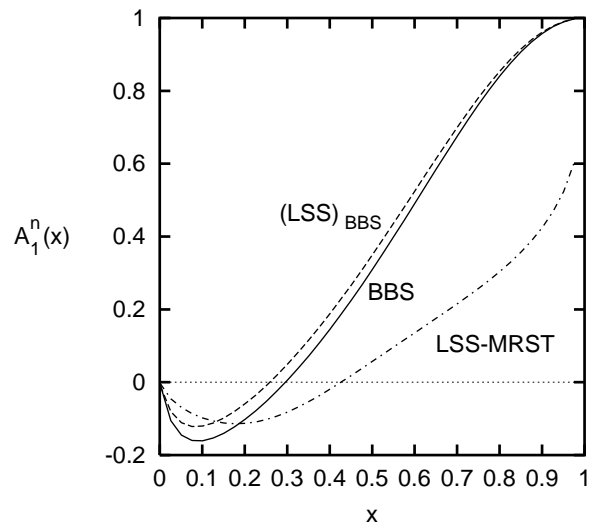


FIG. 11. The neutron longitudinal asymmetry  $A_1^n(x)$  for  $Q^2 \sim 1 - 4 \text{ GeV}^2$ , as obtained by using the BBS and  $(LSS)_{BBS}$  parametrizations (solid and dashed lines respectively), and the LSS-MRST parametrizations (dash-dotted line).